Chapter 3

Performance on Items Within Each Mathematics Content Area

This chapter presents five example items within each of the mathematics content areas, including the performance on each of the items for each of the TIMSS countries. The example items in this chapter were chosen to illustrate the different topics covered within each content area as well as the different performance expectations. The items also were chosen to show the range of item formats used within each content area. To provide some sense of what types of items were answered correctly by higher-performing as compared to lower-performing students, the items show a range of difficulty within each content area. Finally, it should be noted that all these items and others are released for use by the public.¹

The presentation for each of the content areas begins with a brief description of the major topics included in the content area, followed by a series of five tables showing achievement results on example items from that content area. Each table shows the percentages of correct responses on the example item for each of the TIMSS countries at both the third and fourth grades. If the item also was included in the TIMSS mathematics test at the seventh and eighth grades, it is so designated, and the international averages are shown for those grades for purposes of comparison. Each table also presents the example item in its entirety. The correct answer is circled for multiple-choice items and shown in the answer space for short-answer items. For extended-response questions, the answer shown exemplifies the type of student responses that were given full credit. All of the responses shown have been reproduced from students' actual test booklets.

After the tables showing the country-by-country results on each of the items, there is a figure relating achievement on each of the example items to performance on the TIMSS international mathematics scale. This "difficulty map" provides a pictorial representation of achievement on the scale in relation to achievement on the example items for the content area.

WHAT HAVE STUDENTS LEARNED ABOUT WHOLE NUMBERS?

The category of whole numbers included understanding place value through the thousands, ordering and comparing numbers, and solving single- as well as multi-step problems involving the operations of addition, subtraction, and multiplication. As shown by the results in Table 3.1, students in most countries demonstrated a basic understanding of the place value of whole numbers (Example Item 1). Students in the fourth grade, in particular, were successful on this item which required students to select the largest number when given four choices, all with four places but

¹ The IEA retained about one-third of the TIMSS items as secure for possible future use in measuring international trends in mathematics and science achievement. All remaining items are available for general use.

differing in the third-, second-, and first-place values (international average of 86% compared to 76% for the lower grade students.) Differences in the ages at which students begin formal schooling and in curricular emphases appear to be reflected in the results. For example, more than 90% of both the third and fourth graders answered this question correctly in Japan and Korea. In Norway, where students begin school at a somewhat older age and thus have had fewer years of formal schooling, this question was answered correctly by 56% of the lower-grade students and 87% of those in the upper grade.

Table 3.2 presents Example Item 2, asking students to represent an addition fact as a multiplication fact. Students were required to provide a response rather than select an answer in the multiple-choice format. Both 5 x 4 = 20 and 4 x 5 = 20 were considered to be correct responses, as were equivalent written-out statements. As with the results for the item on comparing numbers, students at both grades did well in Hong Kong, Japan, Korea, and Singapore (86% or better). In most countries, however, the results were somewhat lower. As indicated by the international averages of 77% and 63%, respectively, the fourth-grade students tended to perform more than 10 percentage points higher than the third-grade students.

Example Item 3 is a subtraction problem with whole numbers that requires regrouping (see Table 3.3). As noted at the bottom of the table, this item also was part of the mathematics test at seventh and eighth grades. The international averages of 86% at both the seventh and the eighth grades suggest that students in most countries had developed a grasp of how to solve this type of problem prior to the middle school years. In contrast, the results at the third and fourth grades were highly variable. For example, about 90% of the students at both grades answered correctly in Korea and Singapore. Considerable growth between grades was shown in most of the remaining countries. Despite this growth, however, in England and New Zealand fewer than 40% of the fourth graders answered this question correctly.

Example Item 4 asked students to solve an addition number sentence involving whole numbers to different place values (one, two, three, and four places). As shown in Table 3.4, fourth-grade students were more successful than their third-grade counterparts in correctly answering this free-response question (international averages of 63% at fourth grade compared to 44% at third grade). Correct responses included either 700 or its equivalent written out as "seven hundred." In Korea and Singapore, more than 90% of the fourth graders provided the missing value necessary to make the sentence true.

Example Item 5 is the most difficult of the examples shown in the area of whole numbers. Students needed to recognize that compared to 24 multiplied by 18, multiplying 25 by 18 would increase the product by 18. As shown in Table 3.5, on average across countries, fewer than half the fourth graders (45%) and one-third of the third graders (30%) answered this question correctly. Fourth graders in Korea had the best performance (80% correct). Interestingly, increasing the product by 1 (option A) was by far the most popular distracter. Internationally, on average, it was selected by 35% of the fourth graders and 42% of the third graders.

Figure 3.1 presents a pictorial representation of the relationship between performance on the TIMSS international mathematics scale and achievement on the five example items for whole numbers.² The international achievement on each example item is indicated both by the average percent correct across all countries at the third and fourth grades and by the international mathematics scale value, or item difficulty level, for each item. Since the scale was developed based on the performance of students at both grades in all countries, the international scale values apply to both grades and to all countries.

For the figure, the item results have been placed on the scale at the point where students at that level were more likely than not (65% probability) to answer the question correctly. For example, students scoring at or above 530 on the scale were likely to provide a correct response to the item asking for the missing value in the addition number sentence (Example Item 4), and those scoring at or above 614 were likely to respond correctly to the problem about the increase in the product when multiplying 18 by 25 rather than by 24 (Example Item 5). Considering that the international average on the scale was 529 at the fourth grade, however, students achieving at about the level of the international average were unlikely to have answered the latter item correctly. These results, however, varied dramatically by country. Fourth-grade students in Singapore, whose mean achievement was 625, had relatively high probabilities of answering all but the most difficult whole number items correctly. Indeed, this is borne out by Singapore's average percent correct of 83% in this content area at the fourth grade.

² The three-digit item label shown in the lower right corner of the box locating each example item on the item difficulty map refers to the original item identification number used in the student test booklets.

Table 3.1 Whole Numbers

Percent Correct for Example Item 1 Lower and Upper Grades (Third and Fourth Grades*)

	Percen	t Correct	Example 1
Country	Third Grade	Fourth Grade	Choose largest number.
Canada Cyprus Czech Republic ^{†2} England Greece Hong Kong Iceland Iran, Islamic Rep. Ireland Japan Korea New Zealand Norway Portugal [†] Scotland Singapore United States Countries Not Satisfying Guid	77 (2.0) 65 (3.0) 87 (1.6) 69 (3.0) 76 (2.9) 88 (1.5) 60 (3.7) 48 (3.6) 81 (2.1) 91 (1.3) 92 (1.6) 65 (3.3) 56 (3.0) 63 (2.8) 68 (2.3) 85 (1.3) 82 (2.0) relines for Sample P	88 (2.0) 82 (1.8) 93 (1.3) 83 (2.0) 87 (2.6) 95 (1.1) 80 (3.0) 63 (2.9) 88 (2.0) 94 (1.0) 97 (1.0) 83 (2.6) 87 (1.8) 80 (1.9) 85 (1.8) 91 (1.3) 89 (1.5) articipation Rates	Which of these is the largest number? A. 2735 B. 2537 C. 2573 D. 2753
(See Appendix A for Details): Australia Austria ¹ Latvia (LSS) Netherlands Countries Not Meeting Age/Gu Percentage of Older Students	78 (2.3) 81 (3.3) 77 (2.6) 88 (1.9) rade Specifications (; See Appendix A fo	91 (1.5) 95 (1.3) 88 (2.3) 93 (1.7) 'High r Details):	
Slovenia 86 (2.0) 92 (1.5) Countries With Unapproved Sampling Procedures at Classroom Level (See Appendix A for Details):			
Hungary 85 (2.2) 91 (1.5) Unapproved Sampling Procedures at Classroom Level and Not Monting Other Guidelines (See Appendix A for Details);			
¹ Israel Kuwait Thailand	 76 (3.0)	91 (1.8) 47 (1.7) 81 (2.4)	
International Average Percent Correct	76 (0.5)	86 (0.4)	

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade.

Table 3.2Whole Numbers

Percent Correct for Example Item 2 Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 2
Country	Third Grade	Fourth Grade	Addition/multiplication fact.
Canada	58 (2.1)	76 (1.7)	
Cyprus	63 (1.9)	83 (1.5)	
Czech Republic	63 (2.2)	83 (1.6)	
^{†2} England	39 (2.0)	53 (1.9)	Addition Fact
Greece	58 (2.4)	79 (1.9)	4 + 4 + 4 + 4 + 4 = 20
Hong Kong	89 (1.3)	95 (0.9)	
Iceland	38 (2.6)	63 (3.0)	Write this addition fact as a multiplication fact.
Iran, Islamic Rep.	62 (2.0)	73 (1.7)	
Ireland	74 (2.2)	86 (1.6)	$4 \times 5 = 20$
Japan	86 (1.3)	92 (0.8)	
Korea	91 (1.4)	94 (0.9)	
New Zealand	45 (2.7)	67 (2.5)	
Norway	36 (2.8)	66 (2.5)	
Portugal	52 (2.8)	65 (2.3)	
[†] Scotland	51 (2.3)	66 (2.1)	
Singapore	87 (1.1)	90 (0.8)	
United States	67 (2.3)	84 (1.4)	
Countries Not Satisfying Gui See Appendix A for Details):	delines for Sample F	Participation Rates	
Australia	57 (2.4)	71 (1.6)	
Austria	71 (2.6)	82 (1.7)	
¹ Latvia (LSS)	66 (2.5)	81 (2.0)	
Netherlands	78 (1.8)	85 (1.5)	
Countries Not Meeting Age/Grade Specifications (High Percentage of Older Students; See Appendix			
Slovenia	69 (2.3)	86 (1.5)	
Countries With Unapproved Level (See Appendix A for De	Sampling Procedure etails):	s at Classroom	
Hungary	61 (2.3)	80 (1.4)]
Unapproved Sampling Proce Meeting Other Guidelines (Se	dures at Classroom ee Appendix A for De	Level and Not etails):	
¹ Israel		86 (1.4)	
Kuwait		42 (1.9)	
Thailand	54 (2.7)	65 (2.2)	
International Average Percent Correct	63 (0.5)	77 (0.4)	

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade.

Table 3.3 Whole Numbers

Percent Correct for Example Item 3 Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 3
Country	Third Grade	Fourth Grade	Subtraction of 4-digit numbers.
Canada	38 (2.9)	61 (3.1)	
Cyprus	54 (3.3)	79 (2.4)	Subtract: 6000
Czech Republic	61 (2.6)	83 (1.8)	<u>-2369</u>
^{†2} England	23 (2.4)	36 (2.5)	
Greece	46 (3.1)	82 (2.2)	A. 4369
Hong Kong	78 (2.3)	89 (1.5)	D 2740
Iceland	15 (2.3)	47 (3.4)	B. 3/42
Iran, Islamic Rep.	52 (3.6)	62 (3.2)	(C.) 3631
Ireland	49 (3.3)	82 (2.5)	
Japan	73 (1.9)	89 (1.4)	D. 3531
Korea	88 (2.1)	93 (1.6)	
New Zealand	15 (2.3)	30 (3.6)	
Norway	9 (1.9)	60 (3.7)	
Portugal	60 (3.5)	77 (2.3)	
[†] Scotland	29 (2.2)	55 (2.4)	
Singapore	90 (1.0)	91 (1.0)	
United States	52 (3.0)	71 (2.2)	
Countries Not Satisfying Guidelines for Sample Participation Rates (See Appendix A for Details):			
Australia	22 (2.5)	47 (2.2)	
Austria	76 (2.5)	92 (2.0)	
¹ Latvia (LSS)	42 (4.0)	80 (2.8)	
Netherlands	46 (3.1)	86 (2.0)	
Countries Not Meeting Age/Grade Specifications (High Percentage of Older Students; See Appendix A for Details):			
Slovenia	62 (3.4)	86 (1.9)	
Countries With Unapproved Level (See Appendix A for De	Sampling Procedure etails):	es at Classroom	
Hungary	74 (2.5)	91 (1.5)	
Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):			
¹ Israel		71 (3.4)	
Kuwait		46 (2.4)	
Thailand	50 (3.8)	65 (3.0)	
	(/	\/	
	50 (0.6)	71 (0.5)	
International Average			
Percent Correct	Seventh Grade	Eighth Grade	
	86 (0.5)	86 (0.4)	Note: Item also tested at seventh and eighth grades.

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade.

Table 3.4Whole Numbers

Percent Correct for Example Item 4 Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 4
Country	Third Grade	Fourth Grade	Complete number sentence.
Canada	40 (2.4)	65 (2.2)	
Cyprus	40 (2.0)	61 (1.9)	
Czech Republic	54 (2.2)	77 (1.7)	Here is a number sentence.
^{†2} England	28 (2.1)	49 (2.1)	
Greece	35 (2.4)	52 (2.3)	2000 + + 30 + 9 = 2739
Hong Kong	75 (1.5)	82 (1.4)	
Iceland	17 (2.2)	45 (2.6)	What number goes where the \Box is to make this sentence true?
Iran, Islamic Rep.	21 (1.9)	26 (2.3)	
Ireland	40 (2.1)	62 (2.1)	2000
Japan	73 (1.3)	86 (1.2)	7000 [0100]
Korea	81 (1.5)	91 (1.1)	Answer: 30
New Zealand	27 (2.2)	49 (2.3)	- A
Norway	18 (1.7)	45 (2.2)	
Portugal	45 (2.2)	60 (2.3)	7134
[†] Scotland	21 (1.6)	47 (1.9)	
Singapore	86 (1.1)	92 (0.7)	
United States	36 (2.6)	58 (1.6)	
Countries Not Satisfying Guidelines for Sample Participation Rates (See Appendix A for Details):			
Australia	41 (2.6)	61 (1.9)	
Austria	45 (2.6)	61 (2.2)	
¹ Latvia (LSS)	36 (2.6)	63 (2.5)	
Netherlands	53 (3.0)	83 (1.9)	
Countries Not Meeting Age/Grade Specifications (High Percentage of Older Students; See Appendix A for Details):			
Slovenia	60 (2.1)	81 (1.6)	
Countries With Unapproved Level (See Appendix A for D	l Sampling Procedur Details):	res at Classroom	
Hungary	50 (2.3)	76 (1.9)	
Thailand	42 (3.4)	59 (2.5)	
Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):			
¹ Israel		71 (2.1)	
Kuwait		27 (1.5)	
Thailand	42 (3.4)	59 (2.5)	
International Average Percent Correct	44 (0.5)	63 (0.4)	

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent. A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade.

Table 3.5Whole Numbers

Percent Correct for Example Item 5 Lower and Upper Grades (Third and Fourth Grades*)

Third GradeFourth GradeFourth GradeCanada29 (2.4)41 (3.0)Cyprus14 (2.1)38 (3.1)Czech Republic29 (2.7)50 (2.6) 1^2 England27 (2.9)37 (3.0)Greece16 (2.3)29 (3.1)Hong Kong35 (2.6)63 (2.6)Iceland21 (3.2)33 (3.8)Iran, Islamic Rep.16 (2.3)26 (3.2)Ireland29 (2.9)51 (3.2)Japan48 (2.2)59 (2.6)Korea58 (3.1)80 (1.8)New Zealand18 (2.2)28 (3.0)Norway24 (2.4)38 (2.8)Portugal20 (2.6)37 (2.6)Singapore53 (2.4)73 (1.7)United States31 (3.0)46 (2.1)20 (2.6)Countries Not Satisfying Guidelines for Sample Participation Rates (See Appendix A for Details):Australia35 (2.7)41 (2.5)Australia35 (2.7)41 (2.5)	Country	Percent	t Correct	Example 5
Canada Cyprus29 (2.4)41 (3.0)Cyprus14 (2.1)38 (3.1)Czech Republic29 (2.7)50 (2.6) 1^2 England27 (2.9)37 (3.0)Greece16 (2.3)29 (3.1)Hong Kong35 (2.6)63 (2.6)Iceland21 (3.2)33 (3.8)Iran, Islamic Rep.16 (2.3)26 (3.2)Ireland29 (2.9)51 (3.2)Japan48 (2.2)59 (2.6)Korea58 (3.1)80 (1.8)New Zealand18 (2.2)28 (3.0)Norway24 (2.4)38 (2.8)Portugal20 (2.6)38 (3.0)* Scotland30 (2.6)37 (2.6)Singapore53 (2.4)73 (1.7)United States31 (3.0)46 (2.1)Countries Not Satisfying Guidelines for Sample Participation Rates (See Appendix A for Details):41 (2.5)Australia35 (2.7)41 (2.5)Australia35 (2.7)41 (2.5)		Third Grade	Fourth Grade	
Singapore 53 (2.4) 73 (1.7) United States 31 (3.0) 46 (2.1) Countries Not Satisfying Guidelines for Sample Participation Rates (See Appendix A for Details): Australia Australia 35 (2.7) 41 (2.5)	Canada Cyprus Czech Republic ^{†2} England Greece Hong Kong Iceland Iran, Islamic Rep. Ireland Japan Korea New Zealand Norway Portugal [†] Scotland	$\begin{array}{c} 29 (2.4) \\ 14 (2.1) \\ 29 (2.7) \\ 27 (2.9) \\ 16 (2.3) \\ 35 (2.6) \\ 21 (3.2) \\ 16 (2.3) \\ 29 (2.9) \\ 48 (2.2) \\ 58 (3.1) \\ 18 (2.2) \\ 24 (2.4) \\ 20 (2.6) \\ 30 (2.6) \end{array}$	41 (3.0) 38 (3.1) 50 (2.6) 37 (3.0) 29 (3.1) 63 (2.6) 33 (3.8) 26 (3.2) 51 (3.2) 59 (2.6) 80 (1.8) 28 (3.0) 38 (2.8) 38 (3.0) 37 (2.6) 72 (4.7)	25 × 18 is more than 24 × 18. How much more? A. 1 \overrightarrow{B} 18 C. 24 D. 25
Australia 35 (2.7) 41 (2.5) Austria 32 (2.2) 52 (2.2)	United States Countries Not Satisfying Guid (See Appendix A for Details):	31 (3.0)	46 (2.1) Participation Rates	
Austria 28 (2.9) 58 (3.6) ¹ Latvia (LSS) 30 (3.6) 42 (3.7) Netherlands 46 (2.4) 60 (3.0) Countries Not Meeting Age/Grade Specifications (High Percentage of Older Students; See Appendix A for Details):	Australia Austria ¹ Latvia (LSS) Netherlands Countries Not Meeting Age/G Percentage of Older Students	35 (2.7) 28 (2.9) 30 (3.6) 46 (2.4) Grade Specifications s; See Appendix A fo	41 (2.5) 58 (3.6) 42 (3.7) 60 (3.0) (High or Details):	
Slovenia30 (2.4)51 (3.0)Countries With Unapproved Sampling Procedures at Classroom	Slovenia 30 (2.4) 51 (3.0) Countries With Unapproved Sampling Procedures at Classroom			
Level (See Appendix A for Details): Hungary 29 (2.2) 52 (2.9) Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):	Level (See Appendix A for De Hungary Unapproved Sampling Procee	etails): 29 (2.2) dures at Classroom	52 (2.9) Level and Not	
International Average 30 (0.5) 45 (0.6)	¹ Israel Kuwait Thailand International Average	 24 (3.1) 30 (0.5)	47 (3.4) 19 (1.6) 33 (2.3) 45 (0.6)	

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade.

Figure 3.1

International Difficulty Map for Whole Numbers Example Items Lower and Upper Grades (Third and Fourth Grades*)



*Third and fourth grades in most countries; see Table 2 for information about the grades tested in each country.

NOTE: Each item was placed onto the TIMSS international mathematics scale based on students' performance in both grades. Items are shown at the point on the scale where students with that level of proficiency had a 65 percent probability of providing a correct response.

What Have Students Learned About Fractions and Proportionality?

Within the content area of fractions and proportionality, students were asked to recognize the pictorial representation of common fractions and decimal fractions as well as the relationships between common and decimal fractions. Several items involved addition and subtraction with fractions and decimals. This content area also included several word problems that could be solved with proportionality and one item involving scale on a map. As indicated in Chapter 2, the items in this content area tended to be more difficult for students than those in the other content areas. For example, the international average across countries in the content area of whole numbers was 66% compared to an average of 48% in the content area of fractions and proportionality.

In the least difficult of the example items, Example Item 6, students were asked to recognize that five-ninths of the figure was shaded. As shown in Table 3.6, about half the students internationally selected the correct response (61%, on average, at the fourth grade and 42% at the third grade). There was a considerable range of performance on this item, however. For example, more than 90% of the fourth-grade students answered this question correctly in Hong Kong, Korea, and Singapore. Five-fourths (option A) was the most commonly selected wrong answer. On average, it was indicated by 28% of the students at the third grade and 23% at the fourth grade. Presumably, these students took the fraction to represent five shaded squares and four unshaded squares.

Example Item 7, a multiple-choice word problem where students could have used proportional reasoning, asked how much sauce could be made from 15 tomatoes if five tomatoes yielded one-half of a liter of sauce. As shown in Table 3.7, approximately one-half of the fourth- and third-grade students internationally answered this question correctly (53% and 42%, respectively.) The results were generally uniform across countries, with about 40% to 60% of the fourth graders providing correct responses in most countries. Somewhat more than 60% of the fourth graders provided correct responses in Hong Kong (73%), Korea (67%), and the Netherlands (67%). Internationally, about one-fourth of both the third and fourth graders selected the answer of three liters (option D), indicating that they grasped the 1 to 3 ratio, but not the fractional unit of measure.

On Example Item 8, requiring students to recognize that 0.2 represented the shaded part of the figure, the international averages for the correct answer were 40% and 33% at the fourth and third grades, respectively. In comparison, internationally, on average, 44% of the fourth-grade students and 43% of the third-grade students selected 2.8 (option A) as their answer. Similar to the misconception about fractional representation shown in Example 6, in this item, many students chose the representation that seemed to suggest two shaded and eight unshaded parts of the figure.

Example Item 9 is a word problem involving subtraction of decimals. As shown in Table 3.9, this problem was extremely difficult for students in many countries (international averages of 26% at fourth grade and 12% at third grade). Korea and Singapore were the only two countries where the majority of the fourth graders provided the completely correct response of 63.2 together with their calculation of 96.4 - 33.2 or its equivalent. (On average across countries, approximately 10% of the fourth-grade students and 8% of the third-grade students received partial credit for providing the correct answer but not showing an acceptable description or calculation.)

In Example Item 10, students were asked to explain their answers using words and pictures (see Table 3.10). For the first part of the question, students needed to express verbally, symbolically, or pictorially that 20 is twice as much as 10, or that 10 is half of 20. As indicated by the sample response, many students drew diagrams or pictographs to explain why Juanita was right. The percentage of correct responses includes both those students who agreed that Juanita was right as well as the very few students (less than 1% in any country) who provided satisfactory explanations but gave neither a "yes" or "no" answer regarding whether Juanita was right. For the second part of this question, students were to express verbally, symbolically, or pictorially that 10 is not half of 30. Students were to answer "no" to the question of whether Amanda was right, but a correct explanation received full credit when neither a "ves" or "no" answer was given. Both parts of this item were very difficult for students. The international averages for Part A were 10% and 21%, respectively, for third and fourth graders. Thirty percent or more of the fourth graders provided fully correct answers in Australia, England, Japan, Korea, the Netherlands, and Singapore. On part B, the international averages were 6% for third-grade students and 15% for fourth-grade students. Thirty percent or more of the fourth graders provided fully correct responses in Japan, Korea, and Singapore.

The item difficulty map for fractions and proportionality is shown in Figure 3.2. The least difficult items involved whole-number proportional reasoning and recognizing the shaded parts of a rectangle representing a fraction. In contrast, the more difficult items involved decimals, or required students to explain their reasoning through words and diagrams.

Table 3.6 Fractions and Proportionality

Percent Correct for Example Item 6 Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 6
Country	Third Grade	Fourth Grade	Fraction of figure shaded.
Canada Cyprus Czech Republic ¹² England Greece Hong Kong Iceland Iran, Islamic Rep. Ireland Japan Korea New Zealand Norway Portugal [†] Scotland Singapore United States Countries Not Satisfying Gui	43 (3.3) 67 (2.8) 14 (2.2) 31 (2.8) 27 (2.8) 93 (1.4) 18 (3.2) 37 (2.7) 53 (3.2) 78 (2.2) 85 (2.0) 36 (3.2) 10 (2.5) 15 (2.5) 40 (3.0) 92 (1.0) 63 (3.2) delines for Sample F	79 (2.6) 75 (2.7) 43 (2.9) 48 (3.0) 58 (3.9) 96 (0.9) 20 (3.1) 68 (2.7) 79 (2.8) 89 (1.6) 92 (1.4) 50 (3.5) 25 (2.8) 16 (2.3) 66 (2.5) 94 (1.0) 80 (1.6) Participation Rates	Part of the figure is shaded. What fraction of the figure is shaded? A. $\frac{5}{4}$ B. $\frac{4}{5}$ C. $\frac{6}{9}$
Australia Austria ¹ Latvia (LSS) Netherlands <i>Countries Not Meeting Age/G</i> <i>Percentage of Older Students</i>	54 (2.5) 13 (2.1) 18 (2.6) 30 (3.3) Frade Specifications s; See Appendix A fo	71 (2.2) 45 (3.5) 48 (3.2) 70 (2.9) (High or Details):	$\bigcirc \frac{5}{9}$
Slovenia 20 (2.2) 47 (3.2) Countries With Unapproved Sampling Procedures at Classroom			
Hungary Unapproved Sampling Proce Meeting Other Guidelines (S	26 (3.1) edures at Classroom ee Appendix A for D	58 (3.2) Level and Not etails):	
¹ Israel Kuwait Thailand International Average Percent Correct	 51 (3.7) 42 (0.6)	62 (3.4) 21 (1.9) 75 (3.0) 61 (0.5)	

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade.

Table 3.7Fractions and Proportionality

Percent Correct for Example Item 7 Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 7
Country	Third Grade	Fourth Grade	Sauce from 15 tomatoes.
Canada	33 (2.5)	41 (2.3)	
Cyprus	46 (2.8)	51 (3.0)	Mario uses 5 tomatoes to make half a liter of tomato sauce. How much sauce
Czech Republic	50 (2.4)	64 (2.9)	can he make from 15 tomatoes?
^{†2} England	39 (2.9)	51 (3.0)	
Greece	45 (2.8)	50 (3.2)	(A.) A liter and a half
Hong Kong	61 (1.8)	73 (2.7)	P Two liters
Iceland	34 (3.7)	44 (3.8)	B. Two mers
Iran, Islamic Rep.	35 (3.3)	44 (2.7)	C. Two liters and a half
Ireland	41 (2.9)	56 (2.5)	
Japan	37 (2.5)	45 (2.4)	D. Three liters
Korea	53 (2.7)	67 (2.5)	
New Zealand	37 (3.0)	48 (3.6)	
Norway	33 (2.6)	51 (3.0)	
Portugal	37 (2.7)	42 (3.2)	
[†] Scotland	29 (2.4)	46 (2.5)	
Singapore	51 (2.1)	60 (2.2)	
United States	37 (3.3)	43 (2.0)	
Countries Not Satisfying Guidelines for Sample Participation Rates (See Appendix A for Details):			
Australia	49 (2.9)	59 (2.6)	
Austria	42 (3.3)	51 (3.0)	
¹ Latvia (LSS)	37 (3.5)	53 (3.3)	
Netherlands	42 (2.5)	67 (3.2)	
Countries Not Meeting Age/G Percentage of Older Students	arade Specifications s; See Appendix A fo	(High or Details):	
Slovenia	48 (3.2)	61 (3.2)	
Countries With Unapproved Sampling Procedures at Classroom Level (See Appendix A for Details):			
Hungary	52 (2.7)	60 (2.5)	
Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):			
¹ Israel		60 (2.9)	
Kuwait		23 (2.3)	
Thailand	46 (3.5)	57 (2.7)	
International Average Percent Correct	42 (0.6)	53 (0.6)	

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade.

Table 3.8 Fractions and Proportionality

Percent Correct for Example Item 8 Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 8
Country	Third Grade	Fourth Grade	Decimal representing shaded part of figure.
Canada Cyprus Czech Republic ^{†2} England Greece Hong Kong Iceland Iran, Islamic Rep. Ireland Japan Korea New Zealand Norway Portugal [†] Scotland Singapore United States Countries Not Satisfying Gui (See Appendix A for Details): Australia Austria ¹ Latvia (LSS) Netherlands Countries Not Meeting Age/G Percentage of Older Students	35 (2.9) 16 (2.1) 45 (2.5) 38 (3.2) 36 (2.9) 38 (3.3) 21 (3.6) 24 (2.6) 34 (2.9) 67 (2.3) 64 (2.6) 26 (2.9) 28 (3.2) 57 (3.1) 34 (2.4) 23 (1.4) 18 (2.5) delines for Sample F 30 (2.4) 28 (4.1) 26 (2.9) 33 (2.7) Grade Specifications s; See Appendix A for	40 (4.4) 41 (2.5) 31 (2.1) 34 (2.9) 30 (2.9) 73 (1.8) 23 (3.5) 35 (3.3) 48 (3.4) 71 (2.3) 67 (2.7) 25 (2.7) 19 (2.4) 71 (2.7) 34 (2.7) 81 (1.7) 32 (2.5) Participation Rates 40 (2.5) 34 (3.2) 30 (3.2) 32 (2.7) (High or Details):	Which number represents the shaded part of the figure? A. 2.8 B. 0.5
Slovenia 27 (2.5) 29 (3.5) Countries With Unapproved Sampling Procedures at Classroom			
Hungary Unapproved Sampling Proce Meeting Other Guidelines (S	40 (3.3) dures at Classroom ee Appendix A for D	31 (2.7) Level and Not etails):	
¹ Israel Kuwait Thailand International Average Percent Correct	 16 (2.3) 33 (0.6)	28 (3.2) 32 (2.3) 35 (3.2) 40 (0.6)	

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade.

Table 3.9 Fractions and Proportionality

Percent Correct for Example Item 9 Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 9
Country	Third Grade	Fourth Grade	Longest box on shelf.
Canada	9 (1.2)	25 (2.2)	
Cyprus	3 (0.7)	16 (1.4)	Julie put a box on a shelf that is 96.4 centimeters long. The box is 33.2 centime-
Czech Republic	16 (1.6)	38 (1.9)	ters long. What is the longest box she could put on the rest of the shelf? Show
^{†2} England	9 (1.3)	22 (2.0)	all your work.
Greece	8 (1.6)	21 (2.0)	GI 4
Hong Kong	20 (2.6)	32 (2.1)	16,1
Iceland	1 (0.6)	6 (1.1)	-33.2
Iran, Islamic Rep.	1 (0.4)	9 (2.3)	(3)
Ireland	12 (1.5)	29 (1.9)	Answer: \underline{D} , \underline{L}
Japan	26 (1.4)	40 (1.9)	
Korea	34 (1.9)	53 (2.0)	
New Zealand	3 (0.8)	13 (1.9)	
Norway	3 (0.8)	19 (1.8)	
Portugal	8 (1.3)	15 (1.3)	
[†] Scotland	8 (1.2)	27 (2.2)	
Singapore	46 (1.7)	61 (1.6)	
United States	11 (1.3)	32 (1.8)	
Countries Not Satisfying Gui (See Appendix A for Details):	delines for Sample I	Participation Rates	
Australia	8 (1.1)	23 (1.3)	
Austria	7 (1.5)	31 (2.2)	
¹ Latvia (LSS)	8 (1.4)	18 (2.4)	
Netherlands	8 (1.0)	28 (2.2)	
Countries Not Meeting Age/Grade Specifications (High Percentage of Older Students; See Appendix A for Details):			
Slovenia	14 (1.6)	33 (2.5)	
Countries With Unapproved Sampling Procedures at Classroom Level (See Appendix A for Details):			
Hungary	5 (0.9)	13 (1.5)	
Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):			
¹ Israel		25 (1.9)	
Kuwait		5 (0.7)	
Thailand	23 (3.7)	32 (2.7)	
International Average Percent Correct	12 (0.3)	26 (0.4)	

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

Table 3.10 Fractions and Proportionality

Percent Correct for Example Item 10A Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 10A
Country	Third Grade	Fourth Grade	Girl/Boy ratio: Is Juanita right?
Canada Cyprus Czech Republic ^{†2} England Greece Hong Kong Iceland Iran, Islamic Rep. Ireland Japan Korea New Zealand Norway Portugal [†] Scotland Singapore United States Countries Not Satisfying Gui (See Appendix A for Details): Australia Austria ¹ Latvia (LSS) Netherlands Countries Not Meeting Age/C Percentage of Older Students Slovenia Countries With Unapproved Level (See Appendix A for Details)	Third Grade 10 (1.2) 5 (1.0) 9 (1.2) 17 (1.2) 7 (1.6) 3 (0.6) 2 (0.7) 3 (0.7) 12 (1.4) 12 (0.9) 31 (1.7) 14 (1.6) 8 (1.4) 4 (0.8) 11 (1.3) 26 (1.9) 13 (1.6) delines for Sample F 21 (2.0) 3 (0.7) 16 (1.7) Grade Specifications s; See Appendix A for 9 (1.6) Sampling Procedure	Sourth Grade 22 (1.7) 21 (1.6) 25 (1.5) 30 (1.8) 11 (1.4) 11 (2.0) 5 (1.2) 7 (1.2) 26 (1.9) 30 (1.6) 43 (2.4) 25 (2.5) 24 (2.1) 9 (1.1) 26 (1.9) 41 (1.9) 25 (1.5) Participation Rates 34 (1.6) 13 (1.5) 9 (1.5) 38 (2.3) (High or Details): 24 (2.5) rs at Classroom	There are 10 girls and 20 boys in Juanita's class. Juanita said that there is one girl for every two boys. Her friend Amanda said that means $\frac{1}{2}$ of all the students in the class are girls. How many students are there in Juanita's class. Answer: <u>30</u> Is Juanita right? Answer: <u>ves</u> Use words or pictures to explain why. 9 in 15 Boys OF 900000000000000000000000000000000000
Hungary			
Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):			
¹ Israel		19 (1.7)	
Kuwait		7 (1.0)	
Thailand	0 (0.2)	4 (1.3)	
International Average Percent Correct	10 (0.3)	21 (0.4)	

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade. Internationally comparable data are unavailable for Hungary on Example 10A.

Table 3.10 Fractions and Proportionality (Continued)

Percent Correct for Example Item 10B Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 10B
Country	Third Grade	Fourth Grade	Girl/Boy ratio: Is Amanda right?
Canada	3 (0.6)	12 (1.5)	There are 10 girls and 20 hows in Jugnita's class. Jugnita sold that there is one
Cyprus	3 (0.8)	12 (1.2)	There are to grits and 20 boys in Juanta's class. Juanta said that there is one
Czech Republic	5 (1.1)	18 (1.7)	girl for every two boys. Her friend Amanda said that means $\frac{1}{2}$ of all the
^{†2} England	9 (1.0)	20 (1.3)	students in the class are girls
Greece	1 (0.5)	8 (1.2)	
Hong Kong	4 (0.7)	13 (2.0)	How many students are there in Juanita's class. Answer:3
Iceland	0 (0.3)	5 (1.1)	Is Juanita right? Answer: VPS
Iran, Islamic Rep.	0 (0.3)	1 (0.4)	Use words or pictures to explain why.
Ireland	9 (1.5)	25 (1.9)	aint D AGAGG D-
Japan	10 (1.0)	30 (1.5)	Boys BOYS BOAR OD DOD
Korea	20 (1.6)	32 (2.0)	Q 7 7 8988 80000 000000000000000000000000
New Zealand	7 (1.0)	15 (1.6)	(A AMARATA 10 groups of two
Norway	6 (1.1)	15 (1.7)	
Portugal	1 (0.4)	3 (0.6)	Is Amanda right? Answer: V O
[†] Scotland	6 (1.0)	16 (1.7)	Use words and pictures to explain why.
Singapore	22 (1.8)	37 (2.0)	har
United States	6 (1.1)	17 (1.5)	because - in it not logiche
Countries Not Satisfying Guidelines for Sample Participation Rates (See Appendix A for Details):			and it is not 20 hours
Australia	11 (1.2)	21 (1.6)	Do Doys
Austria	2 (1.0)	5 (1.0)	
¹ Latvia (LSS)	1 (0.4)	6 (1.2)	
Netherlands	8 (1.0)	24 (2.1)	
Countries Not Meeting Age/Grade Specifications (High Percentage of Older Students; See Appendix A for Details):			
Slovenia	2 (0.7)	12 (1.7)	
Countries With Unapproved Sampling Procedures at Classroom Level (See Appendix A for Details):			
Hungary			
Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):			
¹ Israel		16 (1.7)	
Kuwait		4 (0.8)	
Thailand	0 (0.2)	4 (1.3)	
International Average Percent Correct	6 (0.2)	15 (0.3)	

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent. A dash (–) indicates data are not available. Israel and Kuwait did not test at the lower grade. Internationally comparable data are unavailable for Hungary on Example 10B.

Figure 3.2

International Difficulty Map for Fractions and Proportionality Example Items Lower and Upper Grades (Third and Fourth Grades*)



*Third and fourth grades in most countries; see Table 2 for information about the grades tested in each country.

NOTE: Each item was placed onto the TIMSS international mathematics scale based on students' performance in both grades. Items are shown at the point on the scale where students with that level of proficiency had a 65 percent probability of providing a correct response.

What Have Students Learned About Measurement, Estimation, and Number Sense?

In the content area of measurement, estimation, and number sense, students were asked to demonstrate their understanding of common measures of length, area, volume, time, the calendar, temperature, and weight. Several questions involved rounding and estimation.

As shown in Example Item 11, students were asked to estimate the length of a pencil in centimeters. The international averages were 77% for fourth graders and 69% for third graders (see Table 3.11). More than half the third- and fourth-grade students in each country answered correctly, except the third graders in the United States (46%), which was the only participating country that does not use the metric system. Because understanding the metric system is a goal of mathematics education in the United States, it used the international version of the measurement items related to the metric system rather than changing these items to reflect the more commonly used measures for length and volume.

Students at both grades also did relatively well on Example Item 12, asking them to select the largest mass given choices ranging from 1 milligram to 1 kilogram. The international averages were 72% for fourth-grade students and 61% for third-grade students. As shown in Table 3.12, 90% or more of the fourth graders answered correctly in Japan, Korea, the Netherlands, and Hungary.

When asked to estimate the total weight of 1000 clothespins each weighing 9.2 grams, students in a number of countries had some difficulty. The international averages on Example Item 13 were 55% for fourth graders and 41% for third graders. As shown in Table 3.13, performance was relatively uniform across countries, ranging at the fourth grade from 74% in the Czech Republic to 38% in Iran, with many countries in the 40% to 60% area.

When asked to apply their knowledge of milliliters in Example Item 14, most students did not recognize that liquid in a teaspoon would be measured in this unit (international averages of 38% and 30%). Perhaps surprisingly, about one-fourth of the students internationally did not seem to recognize that milliliters could not be used to measure thickness. Options C and D were equally popular distracters (both were selected by 23% of the students at grade 4). However, more than 70% of the fourth-grade students in Hong Kong (73%) and Japan (75%) answered this question correctly. The increases from third to fourth grade on this item were relatively small in most countries. One exception, however, was Hong Kong (from 41% to 73%).

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Example Item 15 is a multi-step problem requiring students to apply their understanding of the perimeter of rectangles. As shown by the results in Table 3.15, this item was very difficult for both third- and fourth-grade students. Both grades performed very similarly, as indicated by the international averages of 23% and 21%. The largest increase and the best performance at the fourth grade was by the students in Singapore, from 19% to 46%.

The international difficulty map for the measurement items is presented in Figure 3.3. It indicates that only the students with higher-than-average mathematics scores internationally were likely to demonstrate that they could go beyond a knowledge of basic weights and lengths to apply measurement skills in various situations.

Table 3.11 Measurement, Estimation, and Number Sense

Percent Correct for Example Item 11 Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 11	
Country	Third Grade	Fourth Grade	Estimate pencil length.	
Canada Cyprus Czech Republic ¹² England Greece Hong Kong Iceland Iran, Islamic Rep. Ireland Japan Korea New Zealand Norway Portugal [†] Scotland Singapore United States <i>Countries Not Satisfying Gui</i> <i>(See Appendix A for Details):</i> Australia Austria ¹ Latvia (LSS) Netherlands <i>Countries Not Meeting Age/G</i> <i>Percentage of Older Students</i> Slovenia <i>Countries With Unapproved Students</i> Slovenia	66 (3.3) 71 (3.0) 79 (2.3) 66 (2.8) 58 (2.9) 89 (1.9) 59 (4.3) 51 (2.7) 64 (2.7) 84 (1.6) 84 (2.1) 64 (3.3) 65 (3.3) 50 (3.7) 65 (2.6) 88 (1.1) 46 (2.1) delines for Sample F 68 (2.5) 77 (2.7) 83 (3.1) 63 (2.7) Garade Specifications s; See Appendix A for D Sampling Procedure trails): 61 (2.5) rdures at Classroom ee Appendix A for D	72 (3.0) 86 (1.9) 91 (1.8) 68 (3.0) 63 (4.8) 91 (1.8) 68 (3.7) 66 (2.9) 79 (2.5) 88 (1.4) 85 (2.0) 71 (2.9) 79 (2.4) 66 (3.3) 69 (2.9) 91 (1.0) 55 (2.2) Participation Rates 76 (1.8) 89 (2.0) 85 (2.7) 77 (2.6) (High or Details): 84 (2.3) s at Classroom 78 (2.8) Level and Not etails): 66 (2.8) 67 (2.6) 84 (2.9)	About how long is this picture of a pencil?	
International Average Percent Correct	69 (0.6)	77 (0.5)		

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade.

Table 3.12 Measurement, Estimation, and Number Sense

Percent Correct for Example Item 12 Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 12
Country	Third Grade	Fourth Grade	Choose largest mass.
Canada	52 (3.4)	68 (2.2)	
Cyprus	15 (1.9)	21 (2.2)	Which of these is largest?
Czech Republic	73 (2.8)	81 (2.3)	
^{†2} England	54 (2.5)	62 (2.6)	A. 1 kilogram
Greece	59 (3.1)	71 (2.9)	
Hong Kong	86 (1.8)	89 (1.6)	B. 1 centigram
Iceland	55 (2.9)	71 (3.3)	
Iran, Islamic Rep.	47 (3.1)	53 (3.6)	C. I milligram
Ireland	44 (2.8)	56 (3.6)	D. 1 gram
Japan	90 (1.3)	94 (1.0)	
Korea	81 (2.1)	90 (1.6)	
New Zealand	47 (2.8)	58 (3.3)	
Norway	52 (3.5)	74 (3.1)	
Portugal	48 (3.5)	79 (2.2)	
[†] Scotland	39 (2.7)	55 (2.7)	
Singapore	64 (2.3)	76 (1.7)	
United States	50 (3.3)	61 (2.1)	
Countries Not Satisfying Guid (See Appendix A for Details):	delines for Sample P	Participation Rates	
Australia	63 (3.0)	74 (2.2)	
Austria	84 (2.2)	89 (2.0)	
¹ Latvia (LSS)	64 (3.4)	77 (2.7)	
Netherlands	85 (1.6)	92 (1.7)	
Countries Not Meeting Age/G Percentage of Older Students	rade Specifications ; See Appendix A fo	(High r Details):	
Slovenia	75 (2.9)	89 (2.0)	
Countries With Unapproved S Level (See Appendix A for De	Sampling Procedure tails):	s at Classroom	
Hungary	76 (2.4)	91 (1.7)	
Unapproved Sampling Proce Meeting Other Guidelines (Se	dures at Classroom ee Appendix A for De	Level and Not etails):	
¹ Israel		50 (3.0)	1
Kuwait		67 (2.3)	
Thailand	65 (2.7)	73 (2.9)	
International Average Percent Correct	61 (0.6)	72 (0.5)	

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade.

Table 3.13 Measurement, Estimation, and Number Sense -

Percent Correct for Example Item 13 Lower and Upper Grades (Third and Fourth Grades*)

	Percent	Correct	Example 13
Country	Third Grade	Fourth Grade	Best estimate of clothespin mass.
Canada	42 (2.8)	50 (2.3)	
Cyprus	32 (2.5)	44 (2.7)	The weight (mass) of a clothespin is 9.2 g. Which of these is the best estimate
Czech Republic	50 (2.8)	74 (2.5)	of the total weight (mass) of 1000 clothespins?
^{†2} England	42 (2.8)	47 (2.9)	
Greece	27 (3.3)	55 (2.9)	A. 900 g
Hong Kong	57 (2.5)	71 (2.6)	
Iceland	32 (3.5)	44 (2.9)	B. 9000 g
Iran, Islamic Rep.	29 (2.5)	38 (2.8)	C. 90 000 g
Ireland	40 (3.2)	52 (2.6)	
Japan			D. 900 000 g
Korea	64 (2.5)	67 (2.5)	
New Zealand	39 (3.4)	42 (2.8)	
Norway	27 (2.9)	49 (3.1)	
Portugal	34 (3.4)	43 (2.4)	
[†] Scotland	40 (2.8)	50 (2.7)	
Singapore	55 (2.0)	59 (2.5)	
United States	38 (2.6)	52 (2.6)	
Countries Not Satisfying Gui (See Appendix A for Details):	delines for Sample F	Participation Rates	
Australia	45 (2.5)	53 (2.3)	
Austria	44 (3.6)	65 (2.9)	
¹ Latvia (LSS)	34 (3.2)	57 (3.3)	
Netherlands	45 (2.8)	71 (3.0)	
Countries Not Meeting Age/G Percentage of Older Students	rade Specifications s; See Appendix A fo	(High or Details):	
Slovenia	42 (2.5)	71 (2.6)	
Countries With Unapproved S Level (See Appendix A for De	Sampling Procedure tails):	s at Classroom	
Hungary	48 (3.1)	71 (2.4)	
Unapproved Sampling Proce Meeting Other Guidelines (S	dures at Classroom ee Appendix A for D	Level and Not etails):	
¹ Israel		62 (2.4)	
Kuwait		45 (2.7)	
Thailand	45 (3.3)	50 (3.3)	
International Average Percent Correct	41 (0.6)	55 (0.5)	

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent. A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade. Internationally comparable data are unavailable for Japan on Example 13.

Table 3.14 Measurement, Estimation, and Number Sense

Percent Correct for Example Item 14 Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 14
Country	Third Grade	Fourth Grade	Substance measured in milliliters.
Canada Cyprus Czech Republic ^{†2} England Greece Hong Kong Iceland Iran, Islamic Rep. Ireland Japan Korea New Zealand Norway Portugal [†] Scotland	$\begin{array}{c} 31 \ (2.4) \\ 21 \ (2.3) \\ 28 \ (2.3) \\ 30 \ (2.5) \\ 21 \ (2.6) \\ 41 \ (2.4) \\ 29 \ (3.4) \\ 22 \ (3.0) \\ 28 \ (2.1) \\ 62 \ (2.0) \\ 25 \ (2.2) \\ 22 \ (2.5) \\ 21 \ (2.9) \\ 31 \ (2.3) \\ 27 \ (2.3) \end{array}$	$\begin{array}{c} 31 \ (2.7) \\ 20 \ (2.9) \\ 32 \ (2.5) \\ 37 \ (2.8) \\ 27 \ (3.6) \\ 73 \ (2.3) \\ 29 \ (3.1) \\ 26 \ (3.1) \\ 44 \ (2.8) \\ 75 \ (1.8) \\ 31 \ (2.5) \\ 35 \ (3.6) \\ 24 \ (2.1) \\ 45 \ (3.1) \\ 35 \ (2.2) \end{array}$	 Which of these would most likely be measured in milliliters? A. The amount of liquid in a teaspoon B. The weight (mass) of a pin C. The amount of gasoline in a tank D. The thickness of 10 sheets of paper
Singapore United States Countries Not Satisfying Gui (See Appendix A for Details):	39 (2.3) 33 (2.8) delines for Sample I	45 (1.9) 38 (2.2) Participation Rates	
Australia Austria ¹ Latvia (LSS) Netherlands Countries Not Meeting Age/G Percentage of Older Students	31 (3.0) 43 (3.5) 29 (3.5) 21 (2.0) Grade Specifications s; See Appendix A fo	44 (2.3) 51 (3.0) 42 (3.0) 27 (2.9) (High or Details):	
Slovenia Countries With Unapproved S Level (See Appendix A for De	35 (2.8) Sampling Procedure etails):	45 (3.2) es at Classroom	
Hungary Unapproved Sampling Proce Meeting Other Guidelines (S	41 (2.9) edures at Classroom ee Appendix A for D	55 (2.6) Level and Not etails):	
¹ Israel Kuwait Thailand International Average Percent Correct	 12 (2.1) 30 (0.5)	28 (3.1) 15 (1.6) 22 (2.4) 38 (0.5)	

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade.

Table 3.15 Measurement, Estimation, and Number Sense

Percent Correct for Example Item 15 Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 15
Country	Third Grade	Fourth Grade	Length of rectangle.
Canada	19 (1.9)	23 (2.4)	
Cyprus	25 (2.5)	28 (2.5)	A thin wire 20 centimeters long is formed into a rectangle. If the width of this
Czech Republic	15 (1.8)	16 (1.8)	rectangle is 4 centimeters, what is its length?
^{†2} England	21 (2.6)	29 (3.2)	
Greece	16 (2.2)	16 (2.6)	A. 5 centimeters
Hong Kong	20 (1.8)	29 (1.9)	B 6 centimeters
Iceland	21 (2.7)	12 (1.9)	U. O centimeters
Iran, Islamic Rep.	13 (2.0)	16 (2.2)	C. 12 centimeters
Ireland	25 (2.3)	20 (1.9)	
Japan	33 (2.2)	32 (2.2)	D. 16 centimeters
Korea	37 (2.6)	38 (3.0)	
New Zealand	25 (2.4)	23 (2.3)	
Norway	18 (2.7)	19 (2.2)	
Portugal	19 (2.3)	18 (2.2)	
[†] Scotland	26 (2.6)	24 (1.9)	
Singapore	19 (1.4)	46 (2.0)	
United States	25 (2.7)	23 (1.9)	
Countries Not Satisfying Gui (See Appendix A for Details).	idelines for Sample I :	Participation Rates	
Australia	24 (2.6)	23 (2.1)	
Austria	25 (4.1)	23 (2.3)	
¹ Latvia (LSS)	19 (2.7)	24 (3.1)	
Netherlands	31 (2.8)	35 (3.6)	
Countries Not Meeting Age/C Percentage of Older Student	Grade Specifications s; See Appendix A f	(High or Details):	
Slovenia	11 (1.8)	20 (2.3)	
Countries With Unapproved Level (See Appendix A for De	Sampling Procedure etails):	es at Classroom	
Hungary	13 (1.8)	15 (2.0)	
Unapproved Sampling Proce Meeting Other Guidelines (S	edures at Classroom See Appendix A for D	n Level and Not Details):	
¹ Israel		17 (2.5)	
Kuwait		22 (1.9)	
Thailand	12 (2.1)	15 (2.4)	
International Average Percent Correct	21 (0.5)	23 (0.5)	

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent. A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade.



*Third and fourth grades in most countries; see Table 2 for information about the grades tested in each country.

NOTE: Each item was placed onto the TIMSS international mathematics scale based on students' performance in both grades. Items are shown at the point on the scale where students with that level of proficiency had a 65 percent probability of providing a correct response.

What Have Students Learned About Data Representation, Analysis, and Probability?

Example Items 16 through 20 illustrate the types of items that were asked about data representation, analysis, and probability. Most of the items in this content area required students to read and use data presented in a variety of charts, tables, and graphs. One task asked them to complete a bar graph from tabular data (Example 20). Two questions dealt with the basic concepts underlying probability (Examples 16 and 18).

Internationally, approximately three-fourths of the fourth graders and two-thirds of the third graders correctly answered Example Item 16 (see Table 3.16). More than 80% of the fourth-grade students in many of the countries appeared to understand that the target with the greatest shaded space had the best chance of being hit.

As shown in Table 3.17, students had little difficulty reading basic information from a bar graph of daily cartons of milk sold at a school, even though some simple interpolation was required to determine that 25 cartons of milk were sold on Monday (see Example Item 17A). The results indicate that this activity was familiar to students in nearly all countries, except Iran and Kuwait. The international averages were 75% and 60%, respectively, at the two grades tested. Students had more difficulty with the second part of this free-response item, when they were asked to provide the number of cartons of milk sold all week and to show their work (see Example Item 17B). International averages on this part of the item decreased to 37% for fourth graders and 19% for third graders. To receive full credit, students needed to give the answer of 125 as well as show their calculation or provide a description of the procedure used. The fourth graders in Singapore had the best performance, providing 80% correct responses.

Example Item 18 assessed the area of probability. In general, about one-half of the fourth-grade students appeared to understand that the probability of picking the one red marble was highest for the bag with the fewest number of marbles. Table 3.18 shows that the international averages were 40% and 51% at the third and fourth grades, respectively. Fifty percent or more of the students at both grades answered this question correctly in Japan, the United States, Australia, and the Netherlands. This item also was part of the TIMSS mathematics test given to seventh- and eighth-grade students. In comparison to performance at the primary grades, the international averages were 74% and 78% at the seventh and eighth grades, respectively. Fourth graders in the Netherlands did particularly well on this item, performing at about the international average for seventh grade.

Performance across countries varied on Example Item 19, which required students to interpret information shown in a pictograph (see Table 3.19). Essentially, this free-response question asked students to determine the key for the graph given the total number of cedar and hemlock trees. That is, students needed to communicate that each tree symbol represented 100 trees. The international averages were 49% and 34% at the fourth and third grades, respectively, indicating growth between the two grades in many countries. Most notably, in the Netherlands performance

increased from 30% to 63% correct. Variation in performance ranged from 85% or more of the students in Japan answering correctly at both grades to fewer than 20% answering correctly in Iran and Kuwait.

As shown in Table 3.20, Example Item 20 required students to complete a bar graph of the ages of boys and girls from data presented in a chart. To receive full credit, all four bars needed to be drawn to the appropriate heights. There could be a shading or placement problem in one set of the bars (i.e., for age 9 or age 10). The international averages were 41% at the fourth grade and 24% at the third grade. Seventy percent or more of the fourth graders in Hong Kong (75%), Japan (78%), Korea (83%), and Singapore (74%) received full credit for their bar graphs. The next highest performance was in the United States, where 55% of the fourth graders completed the graph according to the requirements. Internationally, on average, approximately 15% of the students received partial credit for having at least one bar completely correct, or the height of all four bars correct with multiple errors in placement or shading.

The item difficulty map presented in Figure 3.4 indicates that students had some difficulty moving beyond a straightforward reading of data in tables to actually using the data in calculations or to representing the data. Only students performing above the international average were likely to answer such questions correctly (Example [°]highest probability of hitting a target with the largest shaded area. In contrast, fewer than half answered that the probability of picking the one red marble from a bag of marbles is highest for the bag with the fewest number of marbles.

Table 3.16 Data Representation, Analysis, and Probability -

Percent Correct for Example Item 16 Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 16	
Country	Third Grade	Fourth Grade	Chance of hitting shaded region.	
Canada	68 (2.4)	83 (1.9)		
Cyprus	55 (3.2)	68 (2.5)	Samantha drops a stone onto each of these targets. The stone has the best	
Czech Republic	78 (2.2)	82 (1.9)	chance of landing on a shaded space in which target?	
^{†2} England	73 (2.3)	78 (1.9)		
Greece	73 (2.8)	84 (2.0)		
Hong Kong	75 (1.7)	84 (1.9)	A. (B.) C. D.	
Iceland	62 (3.3)	76 (3.2)		
Iran, Islamic Rep.	50 (3.6)	70 (2.7)		
Ireland	66 (2.8)	72 (2.5)		
Japan	89 (1.2)	89 (1.5)		
Korea	81 (2.1)	84 (2.0)		
New Zealand	61 (3.2)	74 (2.8)		
Norway	70 (3.3)	86 (2.3)		
Portugal	41 (3.0)	62 (2.7)		
[†] Scotland	66 (2.6)	73 (2.9)		
Singapore	60 (2.2)	70 (1.8)		
United States	75 (2.5)	83 (1.5)		
Countries Not Satisfying Guid (See Appendix A for Details):	delines for Sample F	Participation Rates		
Australia	71 (3.3)	79 (1.9)		
Austria	71 (3.3)	81 (2.7)		
¹ Latvia (LSS)	72 (3.2)	79 (3.0)		
Netherlands	80 (2.3)	86 (2.4)		
Countries Not Meeting Age/G Percentage of Older Students	rade Specifications s; See Appendix A fo	(High r Details):		
Slovenia	71 (2.3)	84 (2.2)		
Countries With Unapproved S Level (See Appendix A for De	Sampling Procedure tails):	s at Classroom		
Hungary	69 (2.5)	76 (2.3)		
Unapproved Sampling Proce Meeting Other Guidelines (So	dures at Classroom ee Appendix A for De	Level and Not etails):		
¹ Israel		85 (2.7)		
Kuwait		58 (1.9)		
Thailand	69 (3.3)	78 (3.0)		
International Average Percent Correct	69 (0.6)	78 (0.5)		

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade.

Table 3.17 Data Representation, Analysis, and Probability

Percent Correct for Example Item 17A Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 17A
Country	Third Grade	Fourth Grade	Bar graph: cartons sold Monday.
Canada Cyprus Czech Republic ¹² England Greece Hong Kong Iceland Iran, Islamic Rep. Ireland Japan Korea New Zealand Norway	82 (1.3) 49 (2.0) 65 (2.0) 45 (2.5) 65 (2.0) 46 (2.5) 7 (1.3) 62 (2.5) 88 (1.1) 89 (1.3) 57 (2.7) 46 (2.3) 25 (2.5)	93 (0.8) 79 (1.3) 85 (1.4) 62 (2.7) 74 (1.7) 77 (2.0) 12 (1.8) 87 (1.7) 94 (0.7) 96 (0.8) 77 (2.2) 79 (1.8) 57 (2.5)	The graph shows the number of cartons of milk sold each day of a week at a school.
[†] Scotland Singapore United States Countries Not Satisfying Gu. (See Appendix A for Details)	35 (2.5) 63 (2.2) 91 (0.8) 75 (1.4) idelines for Sample	57 (2.5) 83 (1.5) 95 (0.5) 90 (0.9) Participation Rates	How many cartons of milk did the school sell on Monday? Answer:
Australia Austria ¹ Latvia (LSS) Netherlands Countries Not Meeting Age/C Percentage of Older Student	71 (2.3) 60 (2.3) 47 (2.5) 78 (2.0) Grade Specifications s: See Appendix A f	88 (0.9) 78 (1.8) 63 (2.6) 94 (1.2) c (High or Details):	How many cartons of milk did the school sell that week? Show your work. $a \\ c \\ $
Slovenia Countries With Unapproved Level (See Appendix A for Do	57 (2.4) Sampling Procedure etails):	73 (2.1) es at Classroom	Answer: $125 + 35$ 125
Hungary Unapproved Sampling Proce Meeting Other Guidelines (S	49 (2.4) edures at Classroon See Appendix A for E	70 (1.8) The Level and Not Details):	
¹ Israel Kuwait Thailand International Average	 55 (3.2) 60 (0.4)	76 (1.9) 17 (1.5) 82 (2.1) 75 (0.3)	
Percent Correct			

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade. Internationally comparable data are unavailable for England on Example 17.

Table 3.17 Data Representation, Analysis, and Probability (Continued)

Percent Correct for Example Item 17B Lower and Upper Grades (Third and Fourth Grades*)

Country Third Grade Fourth Grade Bar graph: cartons sold for week. Canada 23 (2.2) 46 (2.8) 4 (0.8) 18 (1.7) Cyprus 4 (0.8) 18 (1.7) The graph shows the number of cartons of milk sold each day of a week at a school. ¹² England Greece 14 (1.5) 33 (2.2) Hong Kong 23 (2.0) 38 (2.1) 40 Iceland 4 (1.1) 20 (1.8) Irean, Islamic Rep. 1 (0.4) 4 (1.0) Japan 23 (1.7) 32 (1.9) Korea 54 (2.0) 73 (2.0) 30 (2.6) New Zealand 9 (1.3) 30 (2.6)		Percent Correct		Example 17B
Canada 23 (2.2) 46 (2.8) Cyprus 4 (0.8) 18 (1.7) Czech Republic 26 (1.8) 51 (2.0) 12 England - - Greece 14 (1.5) 33 (2.2) Hong Kong 23 (2.0) 38 (2.1) Iceland 4 (1.1) 20 (1.8) Iran, Islamic Rep. 1 (0.4) 44 (1.0) Japan 23 (1.7) 32 (1.9) Korea 54 (2.0) 73 (2.0) New Zealand 9 (1.3) 30 (2.6)	Country	Third Grade	Fourth Grade	Bar graph: cartons sold for week.
New Zealand 9 (1.3) 30 (2.6)	Canada Cyprus Czech Republic ^{†2} England Greece Hong Kong Iceland Iran, Islamic Rep. Ireland Japan Korea	23 (2.2) 4 (0.8) 26 (1.8) 14 (1.5) 23 (2.0) 4 (1.1) 1 (0.4) 15 (1.8) 23 (1.7) 54 (2.0)	46 (2.8) 18 (1.7) 51 (2.0) - - 33 (2.2) 38 (2.1) 20 (1.8) 4 (1.0) 48 (2.2) 32 (1.9) 73 (2.0)	The graph shows the number of cartons of milk sold each day of a week at a school.
Norway 5 (1.0) 28 (1.9) Portugal 11 (1.6) 26 (2.0) † Scotland 19 (1.7) 43 (2.3) Singapore 65 (1.6) 80 (1.3) United States 28 (2.0) 57 (1.9)	New Zealand Norway Portugal [†] Scotland Singapore United States <i>Countries Not Satisfying Gui</i> <i>(See Appendix A for Details):</i>	9 (1.3) 5 (1.0) 11 (1.6) 19 (1.7) 65 (1.6) 28 (2.0) delines for Sample F	30 (2.6) 28 (1.9) 26 (2.0) 43 (2.3) 80 (1.3) 57 (1.9) Participation Rates	Mon. Tues. Wed. Thurs. Fri. Day How many cartons of milk did the school sell on Monday? Answer:
Australia 13 (1.5) 34 (1.8) Austria 15 (1.9) 38 (3.0) ¹ Latvia (LSS) 18 (2.1) 33 (2.7) Netherlands 25 (1.8) 56 (2.5) Countries Not Meeting Age/Grade Specifications (High Percentage of Older Students; See Appendix A for Details): How many cartons of milk did the school sell that week? Slovenia 30 (2.4) 48 (2.7) 1275 75	Australia Austria ¹ Latvia (LSS) Netherlands <i>Countries Not Meeting Age/G</i> <i>Percentage of Older Students</i> Slovenia	13 (1.5) 15 (1.9) 18 (2.1) 25 (1.8) Grade Specifications s; See Appendix A for 30 (2.4)	34 (1.8) 38 (3.0) 33 (2.7) 56 (2.5) (High or Details): 48 (2.7)	How many cartons of milk did the school sell that week? Show your work. 125 125 75 75
Countries With Unapproved Sampling Procedures at Classroom Level (See Appendix A for Details): Hungary 10 (1.3) 28 (1.9)	Countries With Unapproved S Level (See Appendix A for De Hungary	Sampling Procedure etails): 10 (1.3)	s at Classroom 28 (1.9)	Answer:+ <u>5</u> 125
Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):	Unapproved Sampling Proce Meeting Other Guidelines (S	edures at Classroom ee Appendix A for D	Level and Not etails):	
¹ Israel 40 (2.6) Kuwait 3 (0.8) Thailand 8 (1.7) 23 (2.2) International Average Percent Correct 19 (0.3) 37 (0.4)	¹ Israel Kuwait Thailand International Average Percent Correct	8 (1.7) 19 (0.3)	40 (2.6) 3 (0.8) 23 (2.2) 37 (0.4)	

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade. Internationally comparable data are unavailable for England on Example 17.

Table 3.18 Data Representation, Analysis, and Probability

Percent Correct for Example Item 18 Lower and Upper Grades (Third and Fourth Grades*)

	Percent	Correct	Example 18			
Country	Third Grade	Fourth Grade	Chance of picking red marble.			
Canada Cyprus Czech Republic ¹² England Greece Hong Kong Iceland Iran, Islamic Rep. Ireland Japan Korea New Zealand Norway Portugal [†] Scotland Singapore United States <i>Countries Not Satisfying Guid</i> <i>(See Appendix A for Details):</i> Australia Austria ¹ Latvia (LSS) Netherlands <i>Countries Not Meeting Age/G</i> <i>Percentage of Older Students</i> Slovenia <i>Countries With Unapproved Sampling Proce Meeting Other Guidelines (St</i> ¹ Israel Kuwait Thailand	$\begin{array}{c} 49 (3.0) \\ 20 (2.5) \\ 42 (2.5) \\ 41 (2.5) \\ 21 (2.5) \\ 45 (2.0) \\ 36 (4.2) \\ 16 (2.2) \\ 46 (2.6) \\ 64 (1.9) \\ 36 (2.4) \\ 40 (2.6) \\ 41 (4.1) \\ 26 (2.5) \\ 42 (2.5) \\ 46 (2.2) \\ 54 (2.7) \\ delines for Sample F \\ \hline 52 (4.1) \\ 38 (3.8) \\ 23 (2.6) \\ 56 (2.6) \\ 56 (2.6) \\ \hline rade Specifications \\ s; See Appendix A for Delines at Classroom ee Appendix A for Delines at$	63 (3.0) 32 (2.8) 56 (2.5) 55 (2.9) 30 (3.0) 69 (3.1) 47 (3.7) 17 (2.5) 52 (2.6) 70 (2.4) 39 (3.1) 55 (3.6) 58 (3.1) 30 (2.4) 55 (2.5) 61 (1.9) 68 (1.9) Participation Rates 59 (2.3) 54 (3.6) 42 (3.5) 74 (2.6) (High r Details): 49 (2.7) s at Classroom 61 (3.1) Level and Not etails): 51 (2.7) 31 (2.1) 39 (2.8) 51 (0.6)	There is only one red marble in each of these bags.			
Percent Correct	Seventh Grade 74 (0.5)	Eighth Grade 78 (0.4)	Note: Item also tested at seventh and eighth grades.			

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade.

Table 3.19 Data Representation, Analysis, and Probability

Percent Correct for Example Item 19 Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 19
Country	Third Grade	Fourth Grade	Pictograph of trees.
Canada	32 (2.9)	49 (4.9)	
Cyprus	40 (3.0)	50 (2.9)	The graph shows 500 cedar trees and 150 hemlock trees.
Czech Republic	29 (2.7)	51 (3.4)	
^{†2} England	31 (2.6)	49 (2.6)	
Greece	21 (4.2)	25 (2.6)	
Hong Kong	59 (2.7)	83 (2.1)	Cedar 2 2 2 2 2 2 2
Iceland	44 (4.1)	54 (3.9)	
Iran, Islamic Rep.	9 (1.7)	16 (2.3)	
Ireland	32 (3.1)	62 (3.1)	Hemlock Z Z
Japan	85 (1.6)	90 (1.5)	
Korea	40 (2.4)	63 (2.7)	
New Zealand	26 (3.3)	45 (3.3)	\wedge
Norway	20 (3.1)	40 (3.2)	How many trees does each 2 represent?
Portugal	21 (3.0)	22 (2.7)	
[†] Scotland	35 (2.8)	56 (2.8)	100
Singapore	63 (2.1)	75 (1.8)	Answer:
United States	40 (2.4)	68 (2.2)	
Countries Not Satisfying Gui (See Appendix A for Details):	delines for Sample I	Participation Rates	
Australia	35 (4.2)	51 (2.6)	
Austria	31 (3.1)	45 (3.5)	
¹ Latvia (LSS)	20 (2.9)	31 (3.8)	
Netherlands	30 (2.6)	63 (3.2)	
Countries Not Meeting Age/G Percentage of Older Students	Grade Specifications s; See Appendix A fo	(High or Details):	
Slovenia	33 (2.7)	44 (3.6)	
Countries With Unapproved Level (See Appendix A for De	Sampling Procedure etails):	s at Classroom	
Hungary	24 (2.3)	44 (2.4)	
Unapproved Sampling Proce Meeting Other Guidelines (S	edures at Classroom ee Appendix A for D	Level and Not etails):	
¹ Israel		51 (3.1)	
Kuwait		7 (1.4)	
Thailand	27 (3.5)	48 (3.6)	
International Average Percent Correct	34 (0.6)	49 (0.6)	

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade.

Table 3.20 Data Representation, Analysis, and Probability

Percent Correct for Example Item 20 Lower and Upper Grades (Third and Fourth Grades*)

	Percent	t Correct	Example 20
Country	Third Grade	Fourth Grade	Bar graphs of boys and girls.
Canada	24 (1.7)	46 (2.6)	
Cyprus Czech Republic	17 (1 7)	33 (2.3)	This table shows the ages of the girls and boys in a club.
^{†2} England	28 (2 0)	49 (2.2)	
Greece	13 (1.7)	28 (2.0)	Age Number of Girls Number of Boys
Hong Kong	59 (2.1)	75 (1.9)	8 4 6
Iceland	15 (2.1)	36 (2.9)	
Iran, Islamic Rep.	0 (0.2)	1 (0.5)	8 4
Ireland	19 (2.1)	34 (2.0)	10 6 10
Japan	55 (1.5)	78 (1.3)	
Korea	69 (2.0)	83 (1.5)	Use the information in the table to complete the graph for ages 9 and 10.
New Zealand	26 (2.1)	48 (2.8)	
Norway	8 (1.3)	26 (2.5)	Girls
Portugal	5 (1.0)	13 (1.3)	10 Boys
[†] Scotland	24 (1.7)	45 (2.0)	
Singapore	48 (2.1)	74 (1.6)	8
United States	31 (2.3)	55 (1.7)	
Countries Not Satisfying Gui (See Appendix A for Details):	delines for Sample I	Participation Rates	
Australia	29 (2.1)	50 (1.8)	
Austria	16 (2.1)	39 (3.2)	
¹ Latvia (LSS)	16 (1.8)	31 (3.1)	
Netherlands	18 (1.6)	42 (2.8)	
Countries Not Meeting Age/G Percentage of Older Students	rade Specifications s; See Appendix A fo	(High or Details):	
Slovenia	18 (1.8)	32 (2.2)	Age
Countries With Unapproved S Level (See Appendix A for De	Sampling Procedure etails):	s at Classroom	
Hungary	14 (1.5)	31 (2.0)	
Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):		Level and Not etails):	
¹ Israel		37 (2.4)	
Kuwait		8 (0.9)	
Thailand	20 (3.1)	38 (3.2)	
International Average Percent Correct	24 (0.4)	41 (0.4)	

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade.

Figure 3.4

International Difficulty Map for Data Representation, Analysis, and Probability Example Items – Lower and Upper Grades (Third and Fourth Grades*)



*Third and fourth grades in most countries; see Table 2 for information about the grades tested in each country. NOTE: Each item was placed onto the TIMSS international mathematics scale based on students' performance in both grades. Items are shown at the point on the scale where students with that level of proficiency had a 65 percent probability of providing a correct response.

WHAT HAVE STUDENTS LEARNED ABOUT GEOMETRY?

The 14 items in the area of geometry represented a variety of content topics. For example, students were asked to recognize basic two-dimensional and threedimensional forms, know basic terms and properties, use visualization to identify equivalence between turned figures, and read coordinate points on a grid.

The data in Table 3.21 reveal that students at both grades had a high degree of success in determining that the plane was located at 2, D on the game board grid (Example Item 21). The international averages of correct responses were 88% at the fourth grade and 80% at the third grade. More than 90% of the fourth-grade students responded correctly in many countries.

Example Item 22 asked students to identify which of four rectangles was not divided into four equal parts. Third graders had more difficulty than fourth graders (international average of 60% compared to 73% for fourth graders). However, the data in Table 3.22 reveal that performance was highly variable. For example, more than 80% of the students at both grades answered this item correctly in Korea and Singapore. In comparison, fewer than 40% did so in Iran and Kuwait, indicating that such visualization tasks are more prevalent in the primary grade curriculum in some Asian countries than they are in some Middle Eastern countries.

As shown in Table 3.23, Example Item 23 required students to draw what a cut-out shape would look like when it is opened up and flattened out. Students could depict either the cut-out shape or the remaining piece of paper with the shape cut out, but the majority tended to the former. The international averages were 59% correct at the fourth grade and 45% at the third grade. In the four Asian countries, 80% or more of the fourth graders responded correctly, followed by 76% responding correctly in both England and Scotland.

In Example Item 24, students needed to use their counting and map-reading skills to determine the point where a school was located. As shown in Table 3.24, approximately half were successful internationally (54% of the fourth graders and 43% of the third graders). Most countries showed an increase in performance between the grades that corresponded to the international results.

Table 3.25 presents the results for Example Item 25, which was based on a figure of a solid cube. Students were asked about the number of edges on the cube. Generally, students at both grades found this a difficult task (international averages of 42% at the fourth grade and 35% at the third grade). Approximately one-third of the students at both grades selected 8 (option B) as their answer. At both grades, however, there was quite a range in performance. For example, at the fourth grade, performance ranged from 17% in Iran and Kuwait to 72% in Hong Kong. Also, the degree of growth between the two grades varied substantially from country to country.

The item difficulty map for the geometry items is presented in Figure 3.5. It indicates that third-grade students in particular have difficulty with a variety of visualization tasks. Most students at both grades could read the coordinates on a simple grid.

Table 3.21 Geometry -

Percent Correct for Example Item 21 Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 21	
Country	Third Grade	Fourth Grade	Objects on game board grid.	
Canada	85 (2.3)	93 (2.0)		
Cyprus	67 (2.7)	83 (1.9)	This is a game board.	
Czech Republic	89 (1.8)	96 (1.2)		
^{†2} England	89 (1.6)	94 (1.3)	E	
Greece	70 (2.9)	86 (2.0)		
Hong Kong	81 (2.3)	93 (1.4)		
Iceland	78 (3.0)	93 (1.9)	c	
Iran, Islamic Rep.	51 (2.5)	70 (2.6)		
Ireland	80 (2.4)	92 (1.7)	B	
Japan				
Korea	89 (1.8)	91 (1.6)		
New Zealand	85 (2.6)	93 (1.6)		
Norway	78 (2.8)	88 (2.1)	1 2 3 4 5	
Portugal	66 (2.8)	69 (2.3)	Which object is located at $(2 \text{ D})^2$	
[†] Scotland	89 (1.8)	95 (1.2)	(
Singapore	84 (1.3)	92 (1.2)		
United States	93 (1.2)	97 (0.5)	A. The plane 7	
Countries Not Satisfying Guid (See Appendix A for Details):	delines for Sample P	Participation Rates	B The truck	
Australia	86 (2.6)	94 (1.0)		
Austria	83 (2.5)	90 (2.2)		
¹ Latvia (LSS)	70 (2.9)	85 (2.1)	C. The bus $\begin{bmatrix} c c c c c c c c c c c c c c c c c c $	
Netherlands	92 (1.7)	97 (1.1)		
Countries Not Meeting Age/G Percentage of Older Students	rade Specifications s; See Appendix A fo	(High r Details):	D. The boat	
Slovenia	85 (2.0)	93 (1.7)		
Countries With Unapproved S Level (See Appendix A for De	Sampling Procedure tails):	s at Classroom		
Hungary	78 (2.4)	88 (1.8)		
Unapproved Sampling Proce Meeting Other Guidelines (Se	dures at Classroom ee Appendix A for De	Level and Not etails):		
¹ Israel		80 (2.7)		
Kuwait		61 (2.3)		
Thailand	69 (3.1)	80 (3.1)		
International Average Percent Correct	80 (0.5)	88 (0.4)		

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade. Internationally comparable data are unavailable for Japan on Example 21.

Table 3.22Geometry

Percent Correct for Example Item 22 Lower and Upper Grades (Third and Fourth Grades*)

	Percent	Correct	E
Country	Third Grade	Fourth Grade	Rectangle div
Canada Cyprus Czech Republic ^{†2} England Greece Hong Kong Iceland Iran, Islamic Rep. Ireland Japan Korea New Zealand Norway Portugal [†] Scotland Singapore United States <i>Countries Not Satisfying Guid</i>	69 (2.5) 41 (2.9) 63 (3.1) 64 (2.5) 29 (3.0) 62 (2.8) 52 (3.7) 33 (2.7) 56 (2.6) 72 (2.1) 83 (1.9) 57 (3.4) 38 (3.0) 37 (2.6) 71 (2.7) 83 (1.5) 68 (2.2) delines for Sample F	78 (3.7) 66 (2.5) 82 (2.3) 75 (2.1) 52 (3.7) 78 (3.1) 75 (3.6) 38 (2.4) 83 (2.2) 86 (1.4) 90 (2.0) 69 (3.2) 63 (2.8) 49 (2.8) 77 (2.0) 85 (1.4) 83 (1.5) Participation Rates	Which rectangle is NOT
(See Appendix A for Details): Australia Austria ¹ Latvia (LSS) Netherlands Countries Not Meeting Age/G Percentage of Older Students	70 (2.8) 61 (3.6) 58 (3.7) 78 (2.5) rade Specifications s; See Appendix A fo	84 (1.2) 83 (2.6) 75 (3.1) 89 (2.0) (High r Details):	
Slovenia Countries With Unapproved S Level (See Appendix A for De	72 (2.7) Sampling Procedure tails):	78 (2.4) s at Classroom	
Hungary Unapproved Sampling Proce	55 (2.1) dures at Classroom	78 (2.2) Level and Not	
¹ Israel Kuwait Thailand	 54 (3.5)	72 (2.8) 35 (2.3) 71 (2.1)	
International Average Percent Correct	60 (0.6)	73 (0.5)	

xample 22

vided into four parts.

divided into 4 equal parts?



*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade.

Table 3.23 Geometry

Percent Correct for Example Item 23 Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 23
Country	Third Grade	Fourth Grade	Cut-out shape.
Canada	53 (2.8)	68 (1.8)	
Cyprus	11 (1.4)	35 (2.2)	Craig folded a piece of paper in half and cut out a shape.
Czech Republic	50 (2.2)	63 (1.7)	
^{†2} England	61 (1.8)	76 (1.8)	
Greece	18 (2.1)	28 (2.7)	
Hong Kong	71 (2.2)	81 (1.6)	
Iceland	39 (2.1)	57 (2.7)	fold
Iran, Islamic Rep.	10 (1.3)	15 (1.4)	
Ireland	42 (1.9)	57 (2.3)	
Japan	75 (1.7)	88 (1.0)	
Korea	73 (1.9)	84 (1.4)	Draw a picture to show what the cut-out shape will look like when it is opened
New Zealand	48 (2.7)	64 (2.3)	up and flattened out.
Norway	22 (2.2)	43 (2.7)	\land
Portugal	16 (1.6)	26 (2.2)	
[†] Scotland	60 (2.0)	76 (1.8)	
Singapore	54 (1.8)	87 (0.9)	
United States	45 (1.7)	66 (1.9)	
Countries Not Satisfying Guidelines for Sample Participation Rates (See Appendix A for Details):			
Australia	61 (2.6)	72 (1.5)	
Austria	50 (2.9)	66 (2.2)	
¹ Latvia (LSS)	47 (2.9)	60 (2.9)	
Netherlands	55 (2.1)	71 (2.5)	
Countries Not Meeting Age/Grade Specifications (High Percentage of Older Students; See Appendix A for Details):			
Slovenia	56 (2.0)	68 (2.0)	
Countries With Unapproved Sampling Procedures at Classroom Level (See Appendix A for Details):			
Hungary	36 (2.0)	62 (2.0)	
Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):			
¹ Israel		55 (2.4)	1
Kuwait		7 (1.0)	
Thailand	31 (3.6)	46 (3.3)	
International Average Percent Correct	45 (0.5)	59 (0.4)	

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade.

Table 3.24Geometry

Percent Correct for Example Item 24 Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 24
Country	Third Grade	Fourth Grade	Map of city blocks.
Canada Cyprus Czech Republic ^{†2} England Greece Hong Kong Iceland Iran, Islamic Rep. Ireland Japan Korea New Zealand Norway Portugal [†] Scotland	$\begin{array}{c} 49 \ (3.3) \\ 29 \ (2.6) \\ 50 \ (2.7) \\ 46 \ (3.0) \\ 26 \ (2.8) \\ \hline 55 \ (2.7) \\ 41 \ (3.7) \\ 18 \ (2.7) \\ 43 \ (3.0) \\ 62 \ (2.1) \\ \hline 59 \ (2.9) \\ 47 \ (3.7) \\ 41 \ (3.7) \\ 25 \ (2.9) \\ 54 \ (2.8) \end{array}$	59 (2.6) 41 (3.1) 68 (2.5) 64 (3.0) 36 (3.3) 66 (2.3) 48 (3.3) 18 (1.9) 54 (2.8) 77 (2.0) 68 (2.7) 52 (3.5) 60 (3.2) 34 (2.7) 61 (2.5)	North
Singapore United States Countries Not Satisfying Guid (See Appendix A for Details):	28 (1.6) 52 (2.9) delines for Sample F	48 (2.1) 62 (2.1) Participation Rates	The driver of the delivery truck starts at corner X. He goes 3 blocks east and 2 blocks north to get to the school. On what corner is the school located?
Australia Austria ¹ Latvia (LSS) Netherlands <i>Countries Not Meeting Age/G</i> <i>Percentage of Older Students</i>	47 (2.7) 58 (3.5) 46 (3.8) 44 (2.7) rade Specifications s; See Appendix A fo	59 (2.1) 70 (3.5) 63 (3.4) 63 (3.2) (High r Details):	A. A (B.) B C. C D. D
Slovenia Countries With Unapproved S Level (See Appendix A for De	50 (3.2) Sampling Procedure tails):	63 (2.6) s at Classroom	н Е. Е
Hungary Unapproved Sampling Proce Meeting Other Guidelines (So	48 (3.3) dures at Classroom ee Appendix A for Do	59 (3.1) Level and Not etails):	
¹ Israel Kuwait Thailand	 20 (3.3)	54 (3.2) 24 (2.1) 24 (4.2)	
International Average Percent Correct	43 (0.6)	54 (0.6)	

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade.

Table 3.25 Geometry

Percent Correct for Example Item 25 Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 25
Country	Third Grade	Fourth Grade	Edges of cube.
Canada	50 (3.1)	59 (5.3)	
Cyprus	21 (2.1)	25 (2.4)	This picture shows a cube with one edge marked. How many edges does the
Czech Republic	44 (3.0)	47 (3.0)	cube have altogether?
^{†2} England	30 (2.7)	39 (2.6)	edge
Greece	18 (2.4)	33 (3.0)	
Hong Kong	53 (3.2)	72 (2.0)	
Iceland	19 (3.0)	25 (4.1)	
Iran, Islamic Rep.	11 (1.6)	17 (2.5)	
Ireland	35 (3.3)	43 (3.0)	
Japan	53 (2.3)	59 (2.8)	
Korea	48 (2.7)	31 (2.5)	
New Zealand	32 (3.1)	32 (3.3)	
Norway			A. 0
Portugal	35 (3.0)	62 (2.6)	B. 8
[†] Scotland	39 (2.4)	44 (2.8)	
Singapore	33 (1.9)	44 (2.2)	(C.) 12
United States	24 (2.9)	35 (1.9)	D 24
Countries Not Satisfying Guid (See Appendix A for Details):	delines for Sample F	Participation Rates	D. 24
Australia	44 (3.3)	53 (2.6)	
Austria	33 (4.0)	50 (3.4)	
¹ Latvia (LSS)	29 (3.2)	32 (2.8)	
Netherlands	43 (2.8)	55 (2.9)	
Countries Not Meeting Age/Grade Specifications (High Percentage of Older Students; See Appendix A for Details):			
Slovenia	60 (2.8)	60 (2.7)	
Countries With Unapproved S Level (See Appendix A for De	Sampling Procedure tails):	s at Classroom	
Hungary	32 (2.7)	42 (2.7)	
Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):		Level and Not etails):	
¹ Israel		32 (3.0)	
Kuwait		17 (1.8)	
Thailand	14 (3.2)	32 (3.2)	
International Average Percent Correct	35 (0.6)	42 (0.6)	

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent. A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade. Internationally comparable data are unavailable for Norway on Example 25.



*Third and fourth grades in most countries; see Table 2 for information about the grades tested in each country.

NOTE: Each item was placed onto the TIMSS international mathematics scale based on students' performance in both grades. Items are shown at the point on the scale where students with that level of proficiency had a 65 percent probability of providing a correct response.

What Have Students Learned About Patterns, Relations, and Functions?

The 10 items in this content area involved patterns of numbers and shapes, representations of simple numerical situations, and relationships between sequences of numbers. In Example Item 26, students were asked to identify the same pattern as portrayed by two different sets of shapes. The international averages show substantial growth from the third grade (61%) to the fourth grade (72%), and indicate that by the seventh and eighth grades most students (about 90%) can demonstrate this basic skill (see Table 3.26). The increase between the third and fourth grades was particularly notable in Greece, from 29% to 51%.

Example Item 27 required students to recognize that 4 x 4 was less than 17, and that, thus, four would make the number sentence true. Internationally, growth between grades also was substantial on this item, from 55% to 70% (see Table 3.27). Third-grade students in Korea did particularly well on this item – 88% correct. The next highest performance at the third grade was in Japan (79%), Slovenia (75%), Singapore (73%), and Latvia (72%). Such results indicate that representation of numerical situations is introduced quite early in the curriculum of some countries.

Example Item 28 asked students to work out the relationships among several logic statements in order to determine that Henry is older than Peter. As shown in Table 3.28, results also were relatively stable across countries. However, the international averages of 63% at the fourth grade and 55% at the third grade indicate less than the average increase of 13% between grades shown in Chapter 2.

To receive full credit on Example Item 29, students needed to demonstrate that they understood what to do to get the next number in a subtraction series. For example, they could explain that the numbers were decreasing by 4, provide the next number or numbers in the series, or give any other type of answer that communicated information about the operation involved. As shown in Table 3.29, the international averages were 57% at the fourth grade and 41% at the third grade. There was a great deal of variation across countries, however. For example, at the fourth grade 79% of the students answered correctly in Singapore and Hungary compared to 12% in Kuwait.

Example Item 30 required students to identify the relationship between two columns of numbers. The results in Table 3.30 reveal that, on average across countries, only 39% of the fourth graders and 27% of the third graders determined that you needed to divide the number in Column A by 5 to obtain the number next to it in Column B. Internationally, about one-fourth of both the third and fourth graders answered that you should subtract 8 from the number in Column A (option B). This response applies only to the first pair of numbers.

Figure 3.6 presents the item difficulty map for the example items in the content area of patterns, relations, and functions. The results indicate that students were more likely to be able to recognize simple patterns and relationships than they were to determine the operations underlying the relationships.

Table 3.26 Patterns, Relations, and Functions

Percent Correct for Example Item 26 Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 26
Country	Third Grade	Fourth Grade	Shapes in a pattern.
Canada	68 (2.7)	78 (3.3)	
Cyprus	40 (3.0)	52 (2.6)	These shapes are arranged in a pattern.
Czech Republic	66 (2.3)	80 (2.3)	
^{†2} England	60 (2.3)	75 (2.3)	ΟΔΟΟΔΔΟΟΔΔΔ
Greece	29 (3.3)	51 (3.2)	
Hong Kong	71 (1.9)	82 (1.9)	Which set of shapes is arranged in the same pattern?
Iceland	51 (3.0)	67 (3.6)	a mini an er ampea a minigee in nie amie parenni
Iran, Islamic Rep.	41 (4.0)	54 (3.2)	A. $\star \Box \star \Box \star \star \Box \Box \star \star \Box \Box$
Ireland	63 (2.4)	75 (2.0)	
Japan	87 (1.5)	91 (1.2)	
Korea	89 (1.8)	93 (1.4)	
New Zealand	59 (3.6)	71 (2.6)	D. $\Box \Box \star \star \Box \star \Box \pm \star \Box \star$
Norway	48 (3.3)	65 (3.0)	
Portugal	42 (2.8)	58 (3.0)	
Scotland	59 (2.4)	73 (2.3)	
Singapore	74 (1.8)	85 (1.4)	
United States	67 (2.7)	79 (1.9)	
Countries Not Satisfying Guid (See Appendix A for Details):	lelines for Sample P	articipation Rates	
Australia	64 (4.0)	78 (1.8)	
Austria	72 (3.6)	85 (1.8)	
¹ Latvia (LSS)	75 (2.6)	82 (2.7)	
Netherlands	75 (2.3)	84 (2.3)	
Countries Not Meeting Age/Grade Specifications (High Percentage of Older Students; See Appendix A for Details):		High r Details):	
Slovenia	54 (3.2)	75 (2.7)	
Countries With Unapproved S Level (See Appendix A for Det	Sampling Procedures tails):	at Classroom	
Hungary	63 (2.8)	80 (2.1)	
Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):			
¹ Israel		69 (2.8)	
Kuwait		44 (2.4)	
Thailand	43 (3.6)	55 (2.8)	
	61 (0.6)	72 (0.5)	
International Average Percent Correct	Seventh Grade	Eighth Grade	
	90 (0.4)	92 (0.3)	Note: Item also tested at seventh and eighth grades.

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade.

Table 3.27 Patterns, Relations, and Functions

Percent Correct for Example Item 27 Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 27
Country	Third Grade	Fourth Grade	Make number sentence true.
Canada	54 (2.4)	69 (2.5)	
Cyprus	47 (2.8)	67 (2.3)	Here is a number sentence.
Czech Republic	65 (2.2)	72 (2.6)	
^{†2} England	41 (2.8)	56 (2.8)	4 × 🔲 < 17
Greece	42 (3.3)	66 (2.7)	
Hong Kong	64 (1.8)	83 (1.9)	Which number could go in the \Box to make the sentence true?
Iceland	41 (3.4)	67 (3.2)	
Iran, Islamic Rep.	38 (3.1)	62 (2.7)	(A) 4
Ireland	46 (3.0)	65 (2.6)	
Japan	79 (1.6)	89 (1.4)	В. 5
Korea	88 (1.7)	93 (1.4)	C. 12
New Zealand	43 (3.3)	57 (2.9)	
Norway	36 (3.4)	63 (3.3)	D. 13
Portugal	46 (3.4)	65 (3.1)	
[†] Scotland	47 (3.3)	59 (2.5)	
Singapore	73 (1.8)	87 (1.4)	
United States	58 (3.0)	71 (2.3)	
Countries Not Satisfying Guidelines for Sample Participation Rates (See Appendix A for Details):		Participation Rates	
Australia	46 (3.2)	65 (2.0)	
Austria	57 (3.2)	74 (2.8)	
¹ Latvia (LSS)	72 (3.2)	79 (3.1)	
Netherlands	53 (2.7)	70 (2.9)	
Countries Not Meeting Age/Grade Specifications (High Percentage of Older Students; See Appendix A for Details):			
Slovenia	75 (2.7)	86 (1.6)	
Countries With Unapproved Sampling Procedures at Classroom Level (See Appendix A for Details):		s at Classroom	
Hungary	63 (2.8)	79 (2.2)	
Unapproved Sampling Proce Meeting Other Guidelines (Se	dures at Classroom ee Appendix A for D	Level and Not etails):	
¹ Israel		77 (2.7)	
Kuwait		43 (2.0)	
Thailand	52 (3.9)	56 (3.1)	
International Average Percent Correct	55 (0.6)	70 (0.5)	

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

Table 3.28 Patterns, Relations, and Functions

Percent Correct for Example Item 28 Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 28
Country	Third Grade	Fourth Grade	True statement of ages.
Canada Cyprus Czech Republic ¹² England Greece Hong Kong Iceland Iran, Islamic Rep. Ireland Japan Korea New Zealand Norway Portugal [†] Scotland Singapore United States Countries Not Satisfying Guid (See Appendix A for Details): Australia Austria ¹ Latvia (LSS) Netherlands Countries Not Meeting Age/G Percentage of Older Students Slovenia Countries With Unapproved S Level (See Appendix A for De Hungary Unapproved Sampling Proce Meeting Other Guidelines (So	59 (2.5) 48 (2.7) 52 (2.4) 54 (3.1) 54 (3.8) 65 (2.1) 47 (3.9) 41 (2.7) 61 (2.4) 68 (2.1) 80 (1.9) 57 (3.4) 52 (3.2) 46 (3.0) 58 (2.6) 62 (2.0) 64 (2.4) delines for Sample F 59 (2.8) 52 (3.3) 38 (3.3) 56 (2.5) rade Specifications s; See Appendix A for 60 (3.1) Sampling Procedures tails): 62 (2.9) dures at Classroom se Appendix A for Deliver 59 (2.9)	68 (2.9) 56 (2.9) 62 (2.4) 66 (2.7) 54 (3.0) 74 (2.0) 61 (4.3) 41 (2.7) 66 (2.8) 73 (2.1) 86 (1.8) 64 (2.9) 66 (3.2) 54 (2.8) 65 (2.1) 72 (1.9) 73 (1.7) Participation Rates 70 (2.0) 57 (3.1) 50 (3.3) 66 (2.8) (High r Details): 72 (3.2) s at Classroom 64 (2.7) Level and Not etails): 68 (3.4) 43 (2.7)	 Henry is older than Bill, and Bill is older than Peter. Which statement must be true? ▲ Henry is older than Peter. B. Henry is younger than Peter. C. Henry is the same age as Peter. D. We cannot tell who is oldest from the information.
Thailand International Average	37 (2.7) 55 (0.6)	47 (3.6) 63 (0.5)	
Percent Correct	00 (0.0)	00 (0.0)	

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade.

Table 3.29 Patterns, Relations, and Functions

Percent Correct for Example Item 29 Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 29
Country	Third Grade	Fourth Grade	Next number in pattern.
Canada	38 (2.4)	57 (2.5)	
Cyprus	41 (2.6)	67 (2.1)	These numbers are part of a pattern.
Czech Republic	42 (2.5)	65 (2.2)	
^{†2} England	41 (2.1)	57 (1.8)	50 , 46 , 42 , 38 , 34 ,
Greece	27 (2.2)	42 (2.6)	With the start have to do to another most much and
Hong Kong	43 (2.6)	65 (1.9)	what do you have to do to get the next number?
Iceland	15 (1.9)	27 (3.0)	20
Iran, Islamic Rep.	14 (1.5)	22 (1.9)	Answer:
Ireland	54 (2.8)	74 (1.9)	
Japan	40 (1.5)	58 (1.4)	
Korea	56 (1.9)	74 (1.7)	
New Zealand	34 (1.8)	48 (2.3)	
Norway	20 (2.2)	46 (2.2)	
Portugal	17 (2.1)	30 (2.3)	
[†] Scotland	44 (2.0)	63 (2.2)	
Singapore	73 (1.4)	79 (1.3)	
United States	40 (1.9)	61 (2.0)	
Countries Not Satisfying Guid (See Appendix A for Details):	delines for Sample F	Participation Rates	
Australia	54 (2.1)	72 (1.5)	
Austria	41 (2.8)	64 (2.5)	
¹ Latvia (LSS)	51 (2.7)	70 (2.5)	
Netherlands	58 (2.1)	77 (1.9)	
Countries Not Meeting Age/Grade Specifications (High Percentage of Older Students; See Appendix A for Details):			
Slovenia	46 (2.5)	68 (2.0)	
Countries With Unapproved Sampling Procedures at Classroom Level (See Appendix A for Details):			
Hungary	62 (2.3)	79 (1.7)	
Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):			
¹ Israel		46 (2.9)	
Kuwait		12 (1.0)	
Thailand			
International Average Percent Correct	41 (0.5)	57 (0.4)	

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent. A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade. Internationally comparable data are unavailable for Thailand on Example 29.

Table 3.30 Patterns, Relations, and Functions

Percent Correct for Example Item 30 Lower and Upper Grades (Third and Fourth Grades*)

	Percent Correct		Example 30
Country	Third Grade	Fourth Grade	Operation to get B from A.
Canada Cyprus Czech Republic ¹² England Greece Hong Kong Iceland Iran, Islamic Rep. Ireland Japan Korea New Zealand Norway Portugal [†] Scotland Singapore United States Countries Not Satisfying Guid (See Appendix A for Details):	18 (2.0) 18 (2.9) 38 (2.6) 19 (2.0) 14 (2.0) 13 (2.6) 18 (2.6) 26 (2.8) 36 (2.0) 50 (3.0) 20 (2.6) 23 (3.5) 23 (2.8) 24 (2.1) 45 (2.1) 27 (2.9) clelines for Sample F	38 (2.6) 29 (2.3) 57 (3.0) 35 (2.7) 24 (2.9) 24 (2.9) 29 (3.2) 38 (2.9) 50 (2.1) 70 (2.2) 27 (2.9) 30 (3.5) 32 (2.6) 31 (2.6) 54 (2.2) 32 (2.3) Participation Rates	What do you have to do to each number in Column A to get the number next to it in Column B? Image: Column A Column B 10 2 115 3 3 125 5 5 115 3 125 5 5 115 115 125 115 115 115 115 115
Austria	25 (2.3) 24 (3.0)	41 (3.8)	
Latvia (LSS) Netherlands	40 (3.5) 30 (3.2)	53 (3.4) 41 (3.2)	
Countries Not Meeting Age/G Percentage of Older Students	rade Specifications s; See Appendix A fo	(High r Details):	
Slovenia	34 (2.6)	47 (3.1)	
Countries With Unapproved S Level (See Appendix A for De	Sampling Procedure tails):	s at Classroom	
Hungary	40 (3.0)	56 (2.7)	
Unapproved Sampling Proce Meeting Other Guidelines (Se	dures at Classroom ee Appendix A for De	Level and Not etails):	
¹ Israel		45 (3.5)	
Kuwait		20 (2.1)	
I hailand	26 (1.9)	37 (2.5)	
International Average Percent Correct	27 (0.6)	39 (0.6)	

*Third and fourth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates data are not available. Israel and Kuwait did not test at the lower grade. Internationally comparable data are not available for Hong Kong in Example 30.

Figure 3.6

International Difficulty Map for Patterns, Relations, and Functions Example Items Lower and Upper Grades (Third and Fourth Grades*)



*Third and fourth grades in most countries; see Table 2 for information about the grades tested in each country. NOTE: Each item was placed onto the TIMSS international mathematics scale based on students' performance in both grades. Items are shown at the point on the scale where students with that level of proficiency had a 65 percent probability of providing a correct response.